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MAT—301

THIRD SEMESTER B.E. DEGREE EXAMINATION

ENGINEERING MATHEMATICS—III

(Common to all Branches of B.E. except C.S. and I.S.E.)

Time : Three Hours Maximum : 100 Marks

Answer any five questions.
All questions carry equal marks.

1. (a) Show that $f(z) = z^n$, is analytic for integral values of n, and hence find $f'(z)$. (6 marks)

   (b) Determine the analytic function $f(z) = u + iv$ given that
   $$u = e^{2x}(x \cos 2y - y \sin 2y).$$
   (7 marks)

   (c) Find the bilinear transformation that maps the points $1, i, -1$, respectively onto the points $i, 0, -i$. Under this transformation, find the image of $|z| < 1$. (7 marks)

2. (a) Find the image of the square region bounded by the lines $x = 1$, $x = 2$, $y = 1$, $y = 2$, under the transformation $w = z^2$. Give the rough sketches of the given region and its image. (6 marks)

   (b) Expand $\dfrac{1}{(z-1)(z-2)}$ in the region $|z| < 1$, $1 < |z| < 2$. (7 marks)

   (c) Evaluate $\int_C \dfrac{dz}{(z^2+4)^2}$, C : $|z - i| = 2$, by Cauchy Integral formula. (7 marks)

3. (a) Expand the function $f(x) = x \sin x$, as a Fourier series in $[-\pi, \pi]$, and deduce the result $\dfrac{\pi - 2}{4} = \dfrac{1}{1.3} - \dfrac{1}{3.5} + \dfrac{1}{5.7} - \ldots$ (7 marks)

   (b) Find the Fourier series of $f(x) = 2x - x^2$, in $[0, 3]$. (6 marks)

   (c) Find the Half-range Fourier cosine series of
   $$f(x) = Kx \quad \text{for} \quad 0 \le x \le l/2$$
   $$= K(l - x) \quad \text{for} \quad l/2 \le x \le l.$$
   (7 marks)

4. (a) Find the Fourier transform of the function
   $$f(x) = a^2 - x^2, \quad |x| < a$$
   $$= 0, \quad |x| > a$$
   and hence show that $\displaystyle\int_0^\infty \frac{\sin x - x\cos x}{x^3}\,dx = \frac{\pi}{4}$.
   (6 marks)

Turn over

(b) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$, $a > 0$. (6 marks)

(c) Find the Fourier series to represent $y(x)$ upto the second harmonic, from the following data :—

$x°$:	30	60	90	120	150	180	210	240	270	300	330	360
y :	2.34	3.01	3.68	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

(8 marks)

5. (a) Find the partial differential equation by eliminating the arbitrary constants a and b from $(x - a)^2 + (y - b)^2 + z^2 = 4$. (6 marks)

(b) Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, for which $\frac{\partial z}{\partial y} = -2 \sin y$ when $x = 0$, and $z = 0$ when y is an odd multiple of $\frac{\pi}{2}$. (7 marks)

(c) Find the general solution of $xz\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} = xy$. (7 marks)

6. (a) Under suitable assumptions, find the missing terms in the following table :—

x :	1	2	3	4	5	6	7
u_x :	103.4	97.6	122.9	—	179.0	—	195.8

(6 marks)

(b) Using Lagrange's interpolation formula, find $f(5)$ from the following values :—

x :	1	3	4	6	9
$f(x)$:	− 3	9	30	132	156

(7 marks)

(c) Using Stirling's formula, find y (0.26) from the following table :—

x :	0.10	0.15	0.20	0.25	0.30
$y(x)$:	0.1003	0.1511	0.2027	0.2553	0.3093

(7 marks)

7. (a) By Runge-Kutta method of order 4, solve $y' = 3x + \frac{y}{2}$, with $y(0) = 1$, and hence find $y(0.2)$, taking $h = 0.1$ (6 marks)

(b) Using Simpson's $\frac{3}{8}$th rule, evaluate $\int_0^{0.3} \sqrt{1 - 8x^3}\, dx$, taking 7 ordinates. (7 marks)

(c) Given

x :	3.0	3.2	3.4	3.6
y :	− 14	− 10.032	− 5.296	0.256

Using Milne's method find y (3.8). (Use Corrector formula twice.) (7 marks)

8. (a) Find the Rank of the matrix $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$ by reducing it to Echelon form.

(6 marks)

(b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{bmatrix}$ and find A^{-1}.

(7 marks)

(c) Using Crout's method (LU-decomposition method), solve the system of equations $x + y + z = 1$, $3x + y - 3z = 5$, $x - 2y - 5z = 10$.

(7 marks)

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MAT - 301

THIRD SEMESTER B.E. DEGREE EXAMINATION, AUGUST/SEPTEMBER 2000

ENGINEERING MATHEMATICS—III

(Common to all branches of BE except CS and ISE)

Time : Three Hours | Maximum : 100 Marks

Answer any five questions.
All questions carry equal marks.

1. (a) Show that $f(z) = z + e^z$ is analytic and hence find $f'(z)$. (7 marks)

 (b) Find the analytic function $f(z) = u + iv$ given

$$u = e^{-x}\left\{\left(x^2 - y^2\right)\cos y + 2xy\sin y\right\}.$$

(7 marks)

 (c) Find the Bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = 0, 1, \infty$.

(6 marks)

2. (a) Under the mapping $w = z^2$, find the images of :

 (i) First quadrant of the z-plane.

 (ii) Region bounded by $x = 1, y = 1, x + y = 1$.

(6 marks)

 (b) Evaluate $\int_C \frac{e^{2z}}{(z+1)(z+2)}dz$ where C is the circle $|z| = 3$. (7 marks)

 (c) Expand $f(z) = \frac{z}{(z-1)(2-z)}$ as a Laurent series valid for (i) $|z| < 1$; (ii) $1 < |z| < 2$; and (iii) $|z| > 2$.

(7 marks)

3. (a) Obtain the Fourier series for the function $f(x)$ defined by

$$\begin{aligned} f(x) &= \pi x && \text{in } 0 \le x \le 1 \\ &= \pi(2 - x) && \text{in } 1 \le x \le 2. \end{aligned}$$

(7 marks)

 (b) Obtain the Fourier series in $(-\pi, \pi)$ for $f(x) = x\cos x$. (7 marks)

 (c) Show that the half-range sine series for the function $f(x) = lx - x^2$ in $0 < x < l$ is

$$f(x) = \frac{8l^2}{\pi^3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^3}\sin\left(\frac{2n+1}{l}\right)\pi x.$$

(6 marks)

4. (a) Find the Fourier transform of $f(x) = 1;\ |x| < 1$ and hence evaluate $\int_0^{\infty}\frac{\sin x}{x}dx$.
 $= 0;\ |x| > 1$

(7 marks)

 (b) Show that $xe^{-x^2/2}$ is self reciprocal under the Fourier sine transforms. (7 marks)

Turn over

(c) Obtain the constant term and the coefficient of the first cosine and sine terms in the Fourier expansion of y from the table :

x :	0	1	2	3	4	5
y :	9	18	24	28	26	20

(6 marks)

5. (a) Form the partial differential equation by eliminating the arbitrary function f from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$.

(6 marks)

(b) Solve : $p\left(1+q^2\right) = q\,(z-1)$. (7 marks)

(c) Solve : $x\left(y^2 - z^2\right)p + y\left(z^2 - x^2\right)q = z\left(x^2 - y^2\right)$. (7 marks)

6. (a) Find the interpolating polynomial $f(x)$ satisfying $f(0) = 0$, $f(2) = 4$, $f(4) = 56$, $f(6) = 204$, $f(8) = 496$, $f(10) = 980$ and hence find $f(3)$.

(7 marks)

(b) Use Lagrange's interpolation formula to find y at $x = 10$ given

x :	5	6	9	11
y :	12	13	14	16

(6 marks)

(c) Apply Bessel's formula to find y_{25} given $y_{20} = 2854$, $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$.

(7 marks)

7. (a) Use Taylors series method to find y at $x = 0.1$, given that $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 1$. (Taking terms upto fourth order derivative in the expansion).

(7 marks)

(b) Using Runge-Kutta method of fourth order find $y(0.2)$ for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$ $y(0) = 1$ taking $h = 0.1$.

(7 marks)

(c) Evaluate $\int_0^1 \frac{x\,dx}{1+x^2}$, by using Simpson's 1/3 rule taking six equal strips and hence deduce an approximate value of $\log_e 2$.

(6 marks)

8. (a) Find the rank of the matrix $A = \begin{pmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{pmatrix}$ using elementary row operations.

(7 marks)

(b) Find the eigenvalues and the corresponding eigenvectors for the matrix $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$.

(7 marks)

(c) Apply Gauss-Seidel iteration method to solve the following system of equations :

$$\begin{aligned} 20x + y - 2z &= 17 \\ 3x + 20y - z &= -18 \\ 2x - 3y + 20z &= 25. \end{aligned}$$

(6 marks)

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MAT 301

THIRD SEMESTER B.E. DEGREE EXAMINATION, MARCH 2001

ENGINEERING MATHEMATICS—III

(Common to all branches of B.E. except C.Sc. and I.S.E.)

Time : Three Hours

Maximum : 100 Marks

Answer any five questions.
All questions carry equal marks.

1. (a) Prove that the function $f(z)$ defined by $f(z) = \dfrac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$ $(z \neq 0)$, $f(0) = 0$ is continuous and Cauchy-Riemann equations are satisfied at the origin.

   (b) Show that the function $u = \sin x \cosh y + 2\cos x \sinh y + x^2 - y^2 + 4xy$ is a harmonic function and determine the corresponding analytic function $f(z)$.

   (c) Show that the transformation $w = i\dfrac{1-z}{1+z}$ transforms the circle $|z| = 1$, into the real-axis of the w-plane and the interior of the circle $|z| < 1$, into the upper-half of the w-plane.

(7 + 6 + 7 = 20 marks)

2. (a) State Cauchy's integral theorem. Evaluate $\int_0^{2+i} \bar{z}^2 dz$, along :

   (i) The line $x = 2y$.

   (ii) The real axis upto 2 and then vertically to $2 + i$.

   (b) Find the Laurent series of $\dfrac{3z^2 - 6z + 2}{z^3 - 3z^2 + 2z}$ in the region, (i) $1 < |z| < 2$; (ii) $|z| > 2$.

   (c) State Cauchy's residue theorem and evaluate $\int_C \dfrac{z \cos z}{(z - \pi/2)^3} dz : |z - 1| = 1$.

(7 + 7 + 6 = 20 marks)

3. (a) Find a Fourier series to represent $f(x) = x - x^2$, in the interval $-\pi < x < \pi$.

   (b) Find a half-range cosine series for $f(x) = \begin{cases} x : & 0 \le x \le \pi/2 \\ \pi - x : & \pi/2 \le x \le \pi. \end{cases}$

   (c) Calculate the first two harmonics of the Fourier series for $f(x)$ from the following data :—

x :	30	60	90	120	150	180	210	240	270	300	330	360
$f(x)$:	2.34	3.01	3.68	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

(6 + 7 + 7 = 20 marks)

4. (a) Find the Fourier cosine transform of the function $f(x) = \begin{cases} 4x, & 0 < x < 1 \\ 4 - x, & 1 < x < 4 \\ 0, & x > 4. \end{cases}$

   (b) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1. \end{cases}$

   Hence evaluate $\int_0^{\infty} \dfrac{x\cos x - \sin x}{x^3} \cos\dfrac{x}{2} dx$.

Turn over

(c) If $f(x) = \begin{cases} \pi x, & \text{in } 0 \le x \le 1 \\ \pi(2-x), & \text{in } 1 \le x \le 2. \end{cases}$

Then show that :

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left(\frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + \ldots\ldots\right)$$

(7 + 7 + 6 = 20 marks)

5. (a) Solve $(y^2 + z^2)\,p + x\,(yq - z) = 0$.

(b) Use the method of separation of variables to solve $\frac{\partial z}{\partial x} = 2\frac{\partial z}{\partial y} + z$. Where $z(x, 0) = 6e^{-3x}$

(c) Obtain the solution of heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$;by using the method of separation of variables.

(6 + 6 + 8 = 20 marks)

6. (a) Using suitable assumptions, find the missing terms in the following table —

x :	45	50	55	60	65
u_x :	3.0	—	2.0	—	– 2.4

(b) Use Stirlings formula to find $f(0.88)$ given :

x :	0.2	0.4	0.6	0.8	1.0
$f(x)$:	0.052	0.073	0.102	0.128	0.151

(c) Using Newton's divided difference formula evaluate $f(8)$:

x :	4	5	7	10	11	14
$f(x)$:	38	90	284	890	1205	2026

(6 + 7 + 7 = 20 marks)

7. (a) Evaluate $\int_0^1 \frac{x\,dx}{1+x^2}$ by Weddle's rule taking seven ordinates and hence find $\log_e 2$.

(b) Use modified Euler's method to solve $\frac{dy}{dx} = x + \sqrt{y}$, in the range $0 \le x \le 0.4$ by taking $h = 0.2$. Given that $y = 1$, at $x = 0$.

(c) If $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$ $y(0.2) = 2.040$ and $y(0.3) = 2.090$ find $y(0.4)$ correct to four decimal places. By using Milne's predictor–Corrector method. (Use Corrector formula twice).

(6 + 7 + 7 = 20 marks)

8. (a) Find the rank of the matrix :

$$\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$$

using elementary row operations.

(b) **Solve the system of equa on :**

$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72.$$

By Gauss-Seidel meth to obtain the final solution to three places of decimals.

(c) Find the eigen roots an he eigen v ctor corresponding to the least eigen value of the matrix :

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

(6 + 7 + 7 = 20 marks)

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MAT301

Reg. No.

Third Semester B.E. Degree Examination, August 2001

Common to all branches except C.S. and I.S.

Engineering Mathematics - III

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions.*
All questions carry equal marks.

1. (a) Derive Cauchy - Riemann equations in Cartesian form

   (b) Find the analytic function $f(z) = u + iv$, given that $u - v = e^x(\cos y - \sin y)$

   (c) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = 2, i, -2$ **(7+6+7 Marks)**

2. (a) State and prove Cauchy's theorem

   (b) Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where c is the circle $|Z| = 3$

   (c) Find the Laurent's expansion for $f(z) = \frac{z^2}{(z-1)(z-3)}$ in the region

   i) $1 < |z| < 3$, ii) $|z - 1| < 2$ **(6+7+7 Marks)**

3. (a) Find the Fourier series for the function $f(x) = |x|$ in $-\pi < x < \pi$ and hence deduce that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$

   (b) Obtain the Half Range cosine series for the function $f(x) = \sin\left(\frac{m\pi x}{l}\right)$, where 'm' is a positive integer over the interval $(0, l)$.

   (c) Given the table

x^o	0^o	60^o	120^o	180^o	240^o	300^o
y	7.9	7.2	3.6	0.5	0.9	6.8

   Obtain the Fourier Series neglecting terms higher than first harmonics. **(7+7+6 Marks)**

4. (a) Find the Fourier Transform of

$$f(x) = \begin{cases} 1 - |x| & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$

   and hence deduce that $\int_0^{\infty} \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}$

   (b) Find the inverse Fourier sine transform of $f_s(\alpha) = \frac{1}{\alpha} e^{-a\alpha}$, $a > 0$

   (c) Find the Fourier cosine transform of

$$f(x) = \begin{cases} 4x, & 0 < x < 1 \\ 4 - x, & 1 < x < 4 \\ 0, & x > 4 \end{cases}$$

(7+6+7 Marks)

Contd.... 2

5. (a) Find the Partial differnetial equation of the family of all spheres whose centres lie on the plane $z = 0$ and have a constant radius r.

(b) Solve, $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$

(c) Solve, $x^2p^2 + y^2q^2 = z^2$

(6+7+7 Marks)

6. (a) Find the missing terms in the following table

x	1	2	3	4	5	6	7
u_x	103.4	97.6	122.9	?	179.0	?	195.8

(b) Given

x	0.1	0.2	0.3	0.4	0.5	0.6
f(x)	2.68	3.04	3.38	3.68	3.96	4.21

Find f(0.25), Using Newton's forward interpolation formula.

(c) Using Lagrange's interpolation formula find f(11) from the data.

x	2	5	8	14
y=f(x)	94.8	87.9	81.3	68.7

(6+7+7 Marks)

7. (a) Evaluate $\int_0^1 \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}$th rule. Hence deduce the value of $\log_e 2$.

(b) Using Taylor Series method find y(0.1), given that $\frac{dy}{dx} = x - y^2, y(0) = 1$ considering upto fourth degree terms.

(c) Use fourth order Runge - Kutta method to solve,

$(x + y)\frac{dy}{dx} = 1, y(0.4) = 1,$ at $x = 0.5,$ correct to four decimal places.

(7+6+7 Marks)

(8) (a) Find the Rank of the matrix

$A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ by reducing it to normal form.

(b) Find all the eigen values and the coresponding eigen vectors for the matrix

$A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$

(c) Apply Gauss - Seidel iterative method (upto four iterations) to solve,

$10x + y + z = 12$
$x + 10y + z = 12$
$x + y + 10z = 12$

(6+7+7 Marks)

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Reg. No. [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]

Third Semester B.E. Degree Examination, February 2002

Common to all branches except C.SC and I.SC.

Engineering Mathematics - III

Time: 3 hrs.] 43 [Max.Marks : 100

Note: *i) Answer any FIVE questions.*
ii) All questions carry equal marks.

1. (a) Show that the polar form of Cauchy - Riemann equations are $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}; \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$

   (b) If $f(z)$ is a regular function of Z. Prove that

   $$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$

   (c) Define Bilinear Transformation. Show that the transformation $W = Z + \frac{1}{z}$ transforms from z - plane circles into family of ellipse and radial lines into family of hyperbolae.

   (6+7+7=20 Marks)

2. (a) If $f(z)$ is analytic within and on a closed curve and if 'a' is any point within C then prove that

   $$f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)dz}{z-a}$$

   (b) Find the Laurent Series of $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $1 < |z| < 2$.

   (c) Determine the poles of the function.

   $$f(z = \frac{z^2}{(z-1)^2(z+2)}$$

   and the residue at each pole.

   (6+7+7=20 Marks)

3. (a) Find a Fourier Series to represent $f(x) = e^{x}$ from $x = -\pi$ to $x = \pi$

   (b) Obtain a half range cosine series for

   $$f(x) \begin{cases} = kx & 0 \le x \le \frac{l}{2} \\ = K(l-x), & \frac{l}{2} \le x \le l. \end{cases}$$

   (c) Obtain the constant term and the coefficients of the first sine and cosine terms in the Fourier expansion of y as given below..

x :	0	1	2	3	4	5
y :	9	18	24	28	26	20

   (6+7+7=20 Marks)

4. (a) Express the function $f(x) = \begin{cases} 1 for|x| < 1 \\ 0 for|x| > 1 \end{cases}$ as a Fourier integral and hence evaluate

   $$\int_0^\infty \frac{sin\lambda cos\lambda x}{\lambda}d\lambda$$

   (b) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$

   (c) State and prove convolution theorem for Fourier transform.

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(7+7+6=20

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1. (a) I

(b)

(c)

2. (

5. (a) Obtain the solution of the wave equation.

$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$$

by the method of separation of variables.

(b) Solve $\frac{\partial^2 Z}{\partial x^2} = a^2 Z$ given that $x = 0$, $\frac{\partial z}{\partial x} =$ a Sin y and $\frac{\partial z}{\partial y} = 0$.

(c) Solve the partial differential equation

$$z^2(p^2 + q^2) = x^2 + y^2$$

(7+7+6=20 Marks)

6. (a) Use Newton's Gregary backward interpolation formula to find $F(410)$ if.

x	100	150	200	250	300	350	400
f(x)	10.63	13.03	15.04	16.81	18.42	19.90	21.27

(b) Given the values

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

Using divided difference formula for unequal intervals find f(9).

(c) Evaluate $\int_4^{5.2} log_e x dx$ by using Weddle's rule taking seven ordinates.

(6+7+7=20 Marks)

7. (a) Using Taylor series method find $y(0.1)$, given that $\frac{dy}{dx} = x - y^2$, $y(0) = 1$. considering up to fourth degree terms.

(b) Given $\frac{dy}{dx} = x + y^2, y(0) = 1$, estimate the value of $y(0.2)$ with $h = 0.1$ using Runge Kutta method of order four.

(c) Apply Milne's predictor - corrector formula to compute y(1.4) given that $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and the data is $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$ and $y(1.3) = 2.7514$.

(6+7+7=20 Marks)

8. (a) Solve the following system of equations by Gauss Jordan method.

$$2x + y + 4z = 12$$
$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$

(b) Solve the given set of equations by Gauss Seidel iterative method.

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

(c) Find the largest eigen value and the corresponding eigen vector of the matrix.

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

by power method.

(7+7+6=20 Marks)

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MAT301

Reg. No. [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]

Third Semester B.E. Degree Examination, July/August 2002

Common to all branches except C.Sc and I.Sc.

Engineering Mathematics - III

Time: 3 hrs.] [Max.Marks : 100

Note: *i) Answer any FIVE full questions.*
ii) All questions carry equal marks.

1. (a) If $f(z) = u + iv$ is an analytic function of Z, find
$$f(z)\ if\ u - v = (x-y)(x^2 + 4xy + y^2)$$
   (b) Prove that the transformation $w = SinZ$ maps the families of lines x = constant and y= constant into two families of confocal central conics.
   (c) Find the Bilinear transformation which sends the points $Z = 0, 1, \infty$ into the points $w = -5, -1, 3$ respectively. What are the invariant points in this transformation?
   (6+7+7=20 Marks)

2. (a) If $f(z)$ is analytic within and on a simple closed curve C and a is a point within C, then prove that
$$f(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z-a} dz$$
   (b) Expand $f(z) = \frac{e^{2z}}{(z-1)^3}$ in the Laurent's series about the point $a = 1$.
   (c) State Cauchy's residue theorem and hence evaluate
$$\int_c \frac{Sin\pi z^2 + Cos\pi z^2}{(z-1)^2(z-2)} dz, \quad \text{where } C \text{ is the circle } |z| = 3$$
   (6+7+7=20 Marks)

3. (a) Find a Fourier series to represent $f(x) = x + x^2$ from $x = -\pi$ to $x = \pi$
   (b) Obtain Fourier series for the function $f(x)$ given by
$$f(x) = 1 + \frac{2x}{\pi}, \quad -\pi \le x \le 0$$
$$= 1 - \frac{2x}{\pi}, \quad o \le x \le \pi$$
   (c) Find the half range cosine series for the function $f(x) = (x-1)^2$ in the interval $o < x < 1$
   (6+7+7=20 Marks)

4. (a) Find the Fourier Cosine transform of $f(x) = \frac{1}{1+x^2}$
   (b) Find the Fourier sine transform of $f(x) = e^{-|x|}$ and hence show that $\int_0^\infty \frac{xSinmx}{1+x^2} dx = \frac{\pi}{2} e^{-m} (m > 0)$.
   (c) Compute the first two harmonics of the Fourier series of $f(x)$ given the following table:

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

   (6+7+7=20 Marks)

5. (a) Form the partial differential equation by eliminating the arbitrary function from the relation $Z = yf(\frac{y}{x})$.

Contd.... 2

(b) Solve : $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

(c) Obtain the solution of the heat equation. $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ by the separation of variables method. (6+7+7=20 Marks)

6. (a) From the following table, estimate the number of students who obtained marks between 40 and 45.

Marks	30-40	40-50	50-60	60-70	70-80
Number of Students	31	42	51	35	31

(b) Given the values

x	3	7	11	13	17
f(x)	150	392	1452	2366	5202

Evaluate f(9) using Newton's divided difference formula.

(c) Given

θ	0^0	5^0	10^0	15^0	20^0	25^0	30^0
$tan\theta$	0	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

Using Stirling's formula, show that $tan 16^0 = 0.2867$. (6+7+7=20 Marks)

7. (a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ rd rule taking four equal strips and hence deduce an approximate value of π.

(b) Employ Taylor's method to obtain approximate value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$. Compare the numerical solution obtained with the exact solution $y = 3(e^{2x} - e^x)$.

(c) Given $\frac{dy}{dx} = x^2(1+y)$ and

$y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979$

Evaluate $y(1.4)$ by Adams-Bashforth method. (6+7+7=20 Marks)

8. (a) Find the rank of the matrix using elementary row operation.

$$A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

(b) Test for consistency and solve

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

(c) Find the eigen values and eigen vectors of the matrix.

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

(6+7+7=20 Marks)

** * **

MAT301

Reg. No. [ ]

Third Semester B.E. Degree Examination, January / February 2003

Common to all branches except C.Sc and I.Sc.

Engineering Mathematics - III

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions.*

1. (a) If $f(z) = u(r,\theta) + iv(r\theta)$ is analytic, derive CR equations in polar form. (7 Marks)

   (b) If $f(z) = \frac{x-iy}{x-iy+1}$, show that $f(\bar{z})$ satisfies CR equations. (7 Marks)

   (c) Discuss the transformation $W = e^z$ (6 Marks)

2. (a) Verify Cauchy's theorem for $f(z) = \frac{1}{z}$ over the triangle formed by $(1,2)(1,4),(3,2)$ (7 Marks)

   (b) Evaluate $\int_c \frac{dz}{z^2(z^2+4)}$. Where c is the circle $|Z-2i| = 3$ (7 Marks)

   (c) Obtain the Laurents expansion for the function $\frac{7z-2}{(z+1)z(z-2)}$ valid in $1 < |z+1| < 3$ (6 Marks)

3. (a) Expand $f(x) = 1+2x$ in $-3 < x \leq 0$
   $= 1-2x$ in $0 \leq x < 3$
   as a Fourier series and deduce that $\frac{\pi^2}{8} = \sum \frac{1}{(2n-1)^2}$ (7 Marks)

   (b) Find the half range cosine series for $f(x) = x sin\ x$ in $0 \leq x \leq \pi$ (6 Marks)

   (c) Find the first three terms (a_0, a_1, a_2) in the half range cosine series of $f(x)$ given an

x	0	1	2	3	4	5
f(x)	4	8	15	7	6	2

(7 Marks)

4. (a) Find the Fourier transform of $cos\ ax^2$ (7 Marks)

   (b) Find the Sine transform of $\frac{e^{-ax}}{x}$ (7 Marks)

   (c) State and prove convolution theorem for the Fourier transform. (6 Marks)

5. (a) Solve : $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$ (6 Marks)

   (b) Solve : $(x+y)(p+q)^2 + (x-y)(p-q)^2 = 1$ (6 Marks)

   (c) A tightly streched string with fixed end points at $x = 0, x = l$ is initially in a position $u = a sin^3(\frac{\pi x}{l})$ and released from rest. Find the displacement $u(x,t)$ at any time t.

6. (a) Find the missing values in the following table :

x :	45	50	55	60	65
y :	3.0	-	2.0	-	-2.4

(6 Marks)

Contd.... 2

(b) Using Newton's forward formula, find the value of $f(1.1)$ if

x :	1	1.4	1.8	2.2
$f(x)$:	3.49	4.82	5.96	6.5

(7 Marks)

(c) Given

x	2	2.2	2.4	2.6	2.8
f(x)	-.6	-.45	-.29	-.12	.05

Find x for which $f(x) = 0$, using lagrange interpolation. (7 Marks)

7. (a) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ rule by dividing into 6 equal parts. (6 Marks)

(b) $\frac{dy}{dx} = sinx + cosy, \quad y(2.5) = 0$

Find y at 3.5 in two steps, modify each step 2 times using Eular modified method. (7 Marks)

(c) $\frac{dy}{dx} = \frac{1}{1+x^2} - 2y^2 \quad y(0) = 0$

Find $y(0.5)$ in two steps, using Runge-Kutta 4th order method. (7 Marks)

8. (a) Explain consistency of a system of m linear equations in n variables.

Test whether the system

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$2x + 4y + 6z = 8$$

is consistent. (6 Marks)

(b) Solve the following system of equation by LU method.

$$x_1 + 2x_2 + x_3 = 0$$
$$2x_1 + 2x_2 + 3x_3 = 3$$
$$x_1 + 3x_2 = -2$$

(8 Marks)

(c) Find the Eigen values and Eigen vectors of the matrix.

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

(6 Marks)

** * **

MAT301

USN

Third Semester B.E. Degree Examination, July/August 2003

Common to all branches except C.S and I.S.

Engineering Mathematics - III

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions.*

1. (a) Derive Cauchy - Riemann equations in polar form (7 Marks)

   (b) Determine the analytic function $f(z)$ given $u = e^{2x}[x\cos 2y - y\sin 2y]$ (7 Marks)

   (c) Find the bilinear transformation which maps the points $Z = 1, i, -1$ onto the points $w = 0, 1, \infty$. (6 Marks)

2. (a) If $f(z)$ is analytic within and on a closed curve and if 'a' is any point within C then prove that $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}dz$ (7 Marks)

   (b) Find the Laurent series of $\frac{3z^2-6z+2}{z^3-3z^2+2z}$ in the region i) $1 < |z| < 2$ ii) $|z| < 2$ (7 Marks)

   (c) Find the residue of the function

   $$f(z) = \frac{z}{(z+1)(z-2)^2} \text{ at i) } z = -1 \quad \text{ii) } z = 2$$ (6 Marks)

3. (a) Obtain the Fourier series of the function $f(x) = x\cos x$ in $(-\pi, \pi)$ (7 Marks)

   (b) Find the Fourier series of the function $f(x) = 2x - x^2$ in $0 < x < 3$ (7 Marks)

   (c) Find the cosine half range series of $f(x) = x\sin x$ in $0 < x < \pi$ and deduce that $\frac{\pi-2}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - + \ldots$ (6 Marks)

4. (a) Express y as a Fourier series upto the 2nd harmonics given the following values.

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(7 Marks)

   (b) Find the complex Fourier transform of the function.

   $$f(x) = \begin{cases} 1 & for \quad |x| \leq a \\ 0 & for \quad |x| > a \end{cases}$$

   Hence evaluate $\int_0^\infty \frac{\sin x}{x}dx$ (7 Marks)

   (c) Find the Fourier sine of cosine transforms of $f(x) = e^{\alpha x}, \quad \alpha > 0$ (6 Marks)

5. (a) Solve $y^2 zp \quad x^2(zq + y)$ (6 Marks)

   (b) Solve $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} = 0$ by the method of separation of variables. (7 Marks)

   (c) Obtain the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables. (7 Marks)

Contd.... 2

6. (a) Using suitable assumptions, find the missing terms in the following table using SHIFT OPERATOR method.

x :	0	1	2	3	4	5	6
y :	5	11	22	40	-	140	-

(6 Marks)

(b) Construct an interpolating polynomial in x for the following data using Newton's general interpolation formula.

x :	2	4	5	6	8	10
y :	10	96	196	350	868	1746

(7 Marks)

(c) Use Bessel's formula to find y at $x = 35$ for the following data.

x :	20	30	40	50
y :	512	439	346	243

(7 Marks)

7. (a) Use Simpson's $1/3^{rd}$ rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking seven ordinates. (6 Marks)

(b) Using modified Euler's method to compute $y(0.1)$ given $\frac{dy}{dx} = x^2 + y : y(0) = 1$ by taking $h = 0.05$, perform two iterations in each step. (7 Marks)

(c) Use Adams - Bashforth method to find $y(0.4)$ given $\frac{dy}{dx} = x + y$

x :	0	0.1	0.2	0.3
y :	1	1.1034	1.2428	1.3997

(7 Marks)

8. (a) Test for consistency and solve

$$x + y + z = 6$$
$$x - y + 2z = 5$$
$$3x + y + z = 8$$

(6 Marks)

(b) Find all the eigen values and eigen vector corresponding to numerically smallest eigen value.

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

(7 Marks)

(c) Find the dominant eigen value and the corresponding eigen vector of the matrix.

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

by power method taking the initial eigen vector as $[1, 1, 1]$ (7 Marks)

** * **

NEW SCHEME

MAT31

USN | | | | | | | | | | |

Third Semester B.E Degree Examination, January/February 2004

Engineering Mathematics III

Common to all branches

(New Scheme)

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions choosing at least one question from each part.*
2. All questions carry equal marks.

Part A

1. (a) Compute the real root of the equation $x\log_{10}x - 1.2 = 0$ by the Regula-Falsi method. Carry out three approximations.

(b) The area A of a circle corresponding to the diameter (D) is given below:

D:	80	85	90	95	100
A:	5026	5674	6362	7088	7854

Find the area corresponding to the diameter 105 by using appropriate interpolation formula.

(c) Using Newton's divided difference formula, find f(8), f(15) from the following data:

x:	4	5	7	10	11	13
f(x):	48	100	294	900	1210	2028

(7+7+6=20 Marks)

2. (a) A rod is rotating in a plane. The following table gives the angle θ (radians) through which the rod has turned for various values of the time t (seconds).

t:	0	0.2	0.4	0.6	0.8	1.0	1.2
θ:	0	0.12	0.49	1.12	2.02	3.20	4.67

Calculate the angular velocity and angular accelaration of the rod when $t = 0.6$ seconds. Employ Newton-Gregory forward interpolation formula.

(b) Employ Taylor's series method to find an approximate solution correct to fourth decimal place for the following initial - value problem at $x = 0.1$ and 0.2

$$\frac{dy}{dx} = 2y + 3e^x,\ y(0) = 0$$

(c) Employ Runge-Kutta method of fourth order to solve the equation

$$\frac{dy}{dx} = 3x + \frac{y}{2}, y(0) = 1$$

at $x = 0.2$ taking step length $h = 0.1$ **(7+7+6=20 Marks)**

Contd.... 2

Part B

3. (a) Find the Fourier series for the function $f(x) = |x|$ in $-\pi < x < \pi$. He deduce that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots\dots + \infty$$

(b) Expand $f(x) = e^{-x}$ as a Fourier series in the interval $(-l, l)$

(c) The following Table gives the variations of a periodic current A over a period

t(sec):	0	T/6	T/3	T/2	2T/3	5T/6	T
A(amp):	1.98	1.30	1.05	1.30	−0.88	−0.25	1.98

Show that there is a constant part of 0.75 amp in the current A and also obta the amplitude of the first harmonic.

(7+7+6=20 Ma

4. (a) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - |x|, & |x| \leq 1 \\ 0, & |x| > | \end{cases}$$

Hence deduce that

$$\int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}$$

(b) Find the inverse Fourier sine transform of

$$F_s(\alpha) = \frac{1}{\alpha} e^{-a\alpha}, \alpha > 0$$

(c) Evaluate: i) $z(\cos n\theta)$

ii) $z(a^n \cos n\theta)$

iii) $z(\sin n\theta)$

(7+7+6=20 Mar

Part C

5. (a) Form the partial differential equation of $z = yf(x) + x\phi(y)$, where f and ϕ ar arbitrary functions.

(b) Solve: $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$

(c) Using Charpit's method, solve

$$z^2 = pqxy$$

(7+7+6 Mark

6. (a) Derive the one dimensional heat equation in the usual form $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

(b) Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for $0 < x < 1$, $0 < y < 1$ given that $u(x,0) = u(0,y) = 0$, $u(x,1) = 6x$, $0 < x \leq 1$ and $u(1,y) = 3y, 0 < y < 1$ Divide the region into 9 square meshes.

(c) Solve the wave equation $4\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ given

$$u(0,t) = u(5,t) = 0, \ t \geq 0$$

$$u(x,0) = x(5-x), \ \frac{\partial}{\partial t}u(x,0) = 0, \ 0 < x < 5$$

Find u at $t = 2$ given $h = 1, k = 0.5$

(7+7+6 Marks

Contd.... 3

Part D

7. (a) Find the values of λ for which the system

$$x + y + z = 1$$
$$x + 2y + 4z = \lambda$$
$$x + 4y + 10z = \lambda^2$$

has a solution. Solve it in each case.

(b) Employ Gauss - Seidel iteration method to solve

$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$

carry out 4 iterations, taking the initial approximation to the solution as (1, 0, 3)

(c) Find the numerically largest eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}.$$

by taking the initial approximation to the eigen vector as $[1, 0.8, -0.8]^T$. Perform 5 iterations. **(7+7+6=20 Marks)**

8. (a) Derive Euler's equation in the form

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

(b) Solve the variational equation

$$\delta \int_1^2 \left[x^2 {y'}^2 + 2y^2 + 2xy\right] dx = 0$$

under the conditions $y(1) = y(2) = 0$

(c) State and prove the Brachistochrone problem. **(7+7+6=20 Marks)**

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Part D

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$$x + y + z = 1$$
$$x + 2y + 4z = \lambda$$
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by taking the initial approximation to the eigen vector as $[1, 0.8, -0.8]^T$. Perform 5 iterations.
(7+7+6=20 Marks)

8. (a) Derive Euler's equation in the form

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

(b) Solve the variational equation

$$\delta \int_1^2 \left[x^2 y'^2 + 2y^2 + 2xy\right] dx = 0$$

under the conditions $y(1) = y(2) = 0$

(c) State and prove the Brachistochrone problem. **(7+7+6=20 Marks)**

** * **

(c) Obtain a half-range cosine series for

$$f(x) = kx \text{ in } 0 < x < \frac{\pi}{2}$$
$$= k(\pi - x) \text{ in } \frac{\pi}{2} < x < \pi$$

(6 Marks)

4. (a) Define Fourier Transform of a function. Find the Fourier Transform of the function.

$$f(x) = 1 - x^2 \text{ for } |x| \leq 1$$
$$= 0 \text{ for } |x| > |$$

(7 Marks)

(b) Find the Fourier Cosine Transform of

$$f(x) = \frac{1}{1+x^2}$$

(7 Marks)

(c) Find the z-transforms of the following:

i) n^2 ii) $a^n cos n\theta$

(6 Marks)

PART C

5. (a) Form the partial differential equation by eliminating the function f, given

$$f(x^2 + 2yz, y^2 + 2zx) = 0$$

(7 Marks)

(b) Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ with the usual notation for p, q

(7 Marks)

(c) Solve $2z + p^2 + qy + 2y^2 = 0$ by Charpit's method.

(6 Marks)

6. (a) Derive the one-dimensional heat flow equation in the form $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

(7 Marks)

(b) Solve the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ subject to the boundary conditions

$$u(0,t) = 0 = u(1,t), t \geq 0$$

and the initial conditions

$$u(x,0) = sin\pi x, \frac{\partial u}{\partial t}(x,0) = 0, 0 < x < 1$$

by taking $h = 1/4$ and $k = 1/5$. Carry out second level solutions in the time-scale.

(6 Marks)

(c) Solve the elliptic partial differential equation for the following square mesh using the 5-point difference formula and by setting up the linear equations at the unknown points P, Q, R, S.

		1	2	
1	P		Q	4
2	S		R	5
		4	5	

(6 Marks)

PART D

7. (a) Solve the following system of linear equations by Gauss elimination method.

$$4x + y + z = 4$$
$$x + 4y - 2z = 4$$
$$3x + 2y - 4z = 6$$

(7 Marks)

Contd....

(b) Solve the following system of linear equations by Gauss-Seidel iteration method:

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

Carryout 3 iterations, starting with initial approximation $x_0 = y_0 = z_0 = 0$ (7 Marks)

(c) Find the largest eigen value and the corresponding eigen vector of matrix A, by using the power method by taking initial vector as $[1, 1, 1]^T$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$ (6 Marks)

8. (a) Solve the Euler's equation obtained for the functional $I = \int_{x_0}^{x_1} (1 + x^2 y') dx$ to have an extremum. (7 Marks)

(b) Find the curve passing through the points (x_1, y_1) and (x_2, y_2) to have minimum surface area of revolution, when it is rotated about x-axis. (7 Marks)

(c) Show that the geodesics on a sphere of radius 'a' are its great circles. (6 Marks)

** * **

(b) Find the Fourier cosine transforms of e^{-ax} and xe^{-ax}, where $a > 0$. Deduce that $\int_0^\infty \frac{\cos mx}{x^2+a^2}dx = \frac{\pi}{2a}e^{-am}$ **(7 Marks)**

(c) Find the Z-transforms of i) n^2 ii) $(n+1)^2$ **(7 Marks)**

Part C

5. (a) Form the partial differential equation by eliminating the arbitrary functions ϕ and ψ from the relation $z = \phi(x+ay) + \psi(x-ay)$ where a is a specified constant. **(6 Marks)**

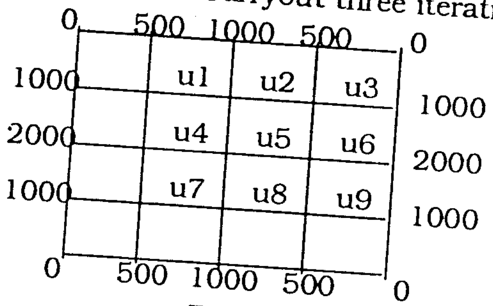
(b) Solve: $(y+z)p + (z+x)q = x+y$ **(7 Marks)**

(c) Using Charpit's method, solve the equation $z^2 = pqxy$ **(7 Marks)**

6. (a) Derive one-dimensional heat equation. **(6 Marks)**

(b) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ with boundary conditions $u(0,t) = u(1,0) = 0,\ t \geq 0$ and initial conditions $u(x,0) = \sin\pi x,\ \frac{\partial u}{\partial t}(x,0) = 0,\ 0 < x < 1$ taking $h = k = 0.2,\ t = 1.0$ **(7 Marks)**

(c) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown: Carryout three iterations.



Part D

7. (a) Find the values of λ and μ for which the system **(7 Marks)**

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

has i) a unique solution ii) infinitely many solutions iii) no solution **(6 Marks)**

(b) Employing the Gauss-seidel method, solve the system,

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

Take $x = 0, y = 0, z = 0$ as an initial approximation to the solution. Carry out five iterations. **(7 Marks)**

(c) Using the Power method, find the dominant eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ starting with the initial vector $[1,\ 1,\ 1]^T$ **(7 Marks)**

8. (a) Obtain the Euler's equation for a variational problem in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}(\frac{\partial f}{\partial y'}) = 0$. Modify this equation when f is independent of y. **(6 Marks)**

(b) Find the extremal of the functional $I = \int_0^{\frac{\pi}{2}} (y^2 - y'^2 - 2y\sin x)dx$ under the end conditions $y(0) = y(\frac{\pi}{2}) = 0$ **(7 Marks)**

(c) Prove that the catenary is the plane curve which when rotated about the x-axis generates a surface of revolution of minimum area. **(7 Marks)**

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NEW SCHEME

MAT31

USN

Third Semester B.E Degree Examination, July/August 2004

Engineering Mathematics III

Common to all branches

(New Scheme)

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer FIVE questions choosing at least one from each of the parts.*

Part A

1. (a) Using Newton-Raphson method, find the root of $x\log_{10}x = 1.2$ near 2.5. Carry out 3 iterations. **(7 Marks)**

(b) Using Newton's forward finite difference interpolation formula, find the number of students who obtained less than 55 marks.

Marks	:	30-40	40-50	50-60	60-70	70-80
No. of students	:	31	42	51	35	31

(7 Marks)

(c) Using Lagrange's interpolation method, find the value of $f(x)$ at $x = 5$, given the values

x:	1	3	4	6
f(x)	3	9	30	132

(6 Marks)

2. (a) Using Simpson's $\frac{1}{3}^{rd}$ rule, evaluate $\int_0^1 \frac{dx}{1+x^2}$ by dividing the interval $(0,1)$ into 6 equal subintervals and hence find the value of π correct to four decimal places. **(7 Marks)**

(b) Solve the differential equation $\frac{dy}{dx} = x^2 + y^2$ given $y(0) = 1$ to find the value of $y(0.1)$ by using Taylor series method of order 4 taking the step length as $h = 0.1$. **(7 Marks)**

(c) Solve the differential equation $\frac{dy}{dx} = 3x + \frac{y}{2}$ given $y(0) = 1$ to find the value of $y(0.1)$ using Runge-Kutta method of order 4, taking the step length $h = 0.1$ **(6 Marks)**

Part B

3. (a) Find the Fourier series of $f(x) = x + x^2$ in the interval $(-\pi, \pi)$ **(7 Marks)**

(b) Obtain the constant term, first two coefficients of cosine terms a_1, a_2 and first two coefficients of sine terms b_1, b_2 in the F.S.expansion of the function given by the following table:

x :	0	1	2	3	4	5
y :	9	18	24	28	26	20

(7 Marks)

Contd.... 2

Contd.... 2

No... 3

(b) **Solve the system of equations:**

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

by Gauss - Seidel method with initial approximation $x_0 = y_0 = z_0 = 0$. Carry out 3 iterations.

(c) Use the power method to find the dominant eigen valve and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ with the initial eigen vector $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. Carryout 5 iterations. **(7+7+6=20 Marks)**

(a) Define a functional and derive Euler's equation in the usual form $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$

(b) Find the curve on which the functional

$$\int_0^1 (y^2 + 12xy)dx \text{ with } y(0) = 0 \text{ and } y(1) = 1 \text{ can be extremised.}$$

(c) Find the curve connecting the points (x_1, y_1) and (x_2, y_2) which when rotated about the x-axis gives a maximum surface area. **(7+7+6=20 Marks)**

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(8)

Pa No... 1

NEW SCHEME

MAT31

USN | | | | | | | | | | |

rd Semester B.E Degree Examination, January/February 2005

Engineering Mathematics III

Common to all branches

me: 3 hrs.] [Max.Marks : 100

Note: 1. *Answer any FIVE full questions choosing at least one question from each part.*

Part A

1. (a) Using the Newton-Raphson method, find an approximate root of the equation $x\log_{10}x = 1.2$ correct to four decimal places that is near 2.5. **(6 Marks)**

(b) From the following table, estimate the number of students who obtained marks between 40 and 45:

Marks	30-40	40-50	50-60	60-70	70-80
No. of Students	31	42	51	35	31

(7 Marks)

(c) Using the Lagrange's formula, find the interpolating polynomial that approximates to the function described by the following table:

x	0	1	2	5
f(x)	2	3	12	147

Hence find $f(3)$. **(7 Marks)**

2. (a) A curve is drawn to pass through the points given by the following table:

x:	1	1.5	2	2.5	3	3.5	4
y:	2	2.4	2.7	2.8	3	2.6	2.1

Using Weddle's rule, estimate the area bounded by the curve, the x-axis and the lines $x = 1, x = 4$. **(6 Marks)**

(b) Using the modified Euler's method, solve the equation $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 1$ in steps of 0.5 at $x = 1$. **(7 Marks)**

(c) Using Milne's predictor - corrector method, find y when $x = 0.8$, given $\frac{dy}{dx} = x - y^2$, $y(0) = 0$ $y(0.2) = 0.02$, $y(0.4) = 0.0795, y(0.6) = 0.1762$. Apply Corrector' formula twice. **(7 Marks)**

Part B

3. (a) Find the half range Fourier Cosine series for the function

$$f(x) = \begin{cases} kx \ in \ 0 \leq x \leq l/2 \\ k(l - x) \quad l/2 < x \leq l \end{cases}$$ where k is a constant. **(6 Marks)**

(b) Obtain the Fourier series of $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ and deduce that

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$ **(7 Marks)**

(c) Compute the constant term and the first two harmonics in the Fourier series of $f(x)$ given by the following table:

x	0	1	2	3	4	5
f(x)	4	8	15	7	6	2

(7 Marks)

4. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x| \text{ for } |x| \leq 1 \\ 0 \text{ for } |x| > 1 \end{cases}$ **(6 Marks)**

Contd.... 2

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NEW SCHEME

MAT31

USN | | | | | | | | | | |

d Semester B.E Degree Examination, July/August 2005

Common to all branches

Engineering Mathematics III

hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions choosing at least one question from each part.*

Part A

. (a) Explain Newton-Raphson method for solving the equation $f(x) = 0$. Use it to find the real root of the equation $xe^x = 2$, correct to three decimal places.

(b) A function y = f(x) is given by the following table

x :	1	1.2	1.4	1.6	1.8	2.0
y :	0.0	0.128	0.544	1.296	2.432	4.00

Find the approximate valves of f(1.1) and $f'(1.1)$ by suitable interpolation formula.

(c) Determine $f(x)$ as a polynomial in x for the following data using Newton's divided difference formula:

x:	−4	−1	0	2	5
y :	1245	33	5	9	1335

(7+7+6=20 Marks)

(a) Evaluate $\int_0^5 \frac{dx}{4x+5}$ by using the Simpson's $\frac{1}{3}$rd rule, taking 10 equal parts. Hence find log 5.

(b) Using modified Euler's method find $y(0.2)$ given $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$, taking $h = 0.1$ (carryout 2 iterations at each stage).

(c) Apply Milne's predictor - corrector formulae to compute $y(0.8)$ given that $\frac{dy}{dx} = x - y^2$ and $y(0) = 0$, $y(0.2) = 0.02$; $y(0.4) = 0.0795$ and $y(0.6) = 0.1762$ (use corrector formula twice)

(7+7+6=20 Marks)

PART - B

(a) Expand the function $f(x) = x(2\pi - x)$ in Fourier series over the interval $(0, 2\pi)$. Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

Contd.... 2

(b) Compute the constant term and the first two harmonics in the Fourier series of f(x) given by the following table:

x :	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(x)$:	1.0	1.4	1.9	1.7	1.5	1.2	1.00

(c) Find the half-range cosine series of $f(x) = (x - 1)^2$ in $0 < x < 1$ (7+7+6=20 M

4. (a) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2; & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$$

Hence evaluate

$$\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx$$

(b) Find the Fourier cosine transform of $f(x)$

$$f(x) = \begin{cases} x & ; \quad 0 < x < 1 \\ 2 - x & ; \quad 1 < x < 2 \\ 0 & ; \quad x > 2 \end{cases}$$

(c) Find Z-transforms of $\cos n\theta$ sin $(n\theta)$. (7+7+6=20 M

PART - C

5. (a) Form the partial differential equation by eliminating the arbitrary functi ϕ and Ψ from the relation $Z = \phi(x + ay) = \Phi\ (x - y)$ where 'a' is a specil constant.

(b) Solve $(y^2 + z^2)p + x(yq - z) = 0$

c) Solve $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} = 0$ by the method of separation of variables.

(7+7+6+20 Ma

6. (a) Derive one dimensional wave equation in the usual form $\frac{\partial^2 u}{\partial t^2} = C^2\frac{\partial^2 u}{\partial x^2}$.

(b) Solve numerically the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0,t) = 0 = u(1,t)$ $(t \ge 0)$ and $u(x,0) = \sin \pi x. 0 \le x \le 1$. Carry c computations for two levels, taking $h = \frac{1}{3}$ and $k = \frac{1}{36}$.

(c) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial yz} = 0$ for $0 < x < 1,\ 0 < y < 1$ given th $u(x,0) = u(0,y) = 0,\ u(x,1) = 6x$ for $0 \le x \le 1$ and $u(1,y) = 3y$ for $0 < y <$

(7+7+6=20 Ma

PART - D

7. (a) Show that the system of equations

$$x + y + z = 4$$
$$2x + y - z = 1$$
$$x - y + 2z = 2$$

is consistent and hence find the solution

Contd...

(b) Solve th
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(c) Use the p
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vector

(a) Define a
$\frac{\partial F}{\partial y}$

(b) Find the
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(c) Find the
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MAT31

USN

NEW SCHEME

Third Semester B.E. Degree Examination, July 2006
Common to All Branches
Engineering Mathematics – III

Time: 3 hrs.] [Max. Marks:100

Note: *1. Answer any FIVE full questions choosing at least one question from each part.*

PART-A

1 a. Using the Newton – Raphson method, find the root that lies near x=4.5, of the equation tanx=x, correct to four decimal places. (Here x is in radians). **(07 Marks)**

b. Use Newton's divided difference formula and find f(4), given the data **(07 Marks)**

x	0	2	3	6
f(x)	-4	2	14	158

c. If y(1)=3, y(3)=9, y(4)=30, y(6)=132, find Lagrange's interpolation polynomial. **(06 Marks)**

2 a. Using Simpson's 1/3rd rule, evaluate $\int_0^1 e^{-x^2}dx$ by dividing the interval (0,1) into 10 sub intervals, (h=0.1). **(07 Marks)**

b. Using the modified Euler's method, solve the initial value problem $\frac{dy}{dx}=\log(x+y)$, y(1)=2 at the points x=1.2 and 1.4, take h=0.2. **(07 Marks)**

c. Apply Runge-Kutta method of fourth order, and find an approximate value of y at x=0.1, given that $\frac{dy}{dx}=3e^x+2y$ with y(0)=0 and h=0.1. **(06 Marks)**

PART-B

3 a. Find the Fourier series for the function $f(x)=\frac{\pi-x}{2}$ in $0<x<2\pi$. Hence deduce that $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\ldots\ldots\ldots\ldots$ **(07 Marks)**

b. Expand $f(x)=2x-x^2$ as a Fourier series in $0\le x\le 2$. **(07 Marks)**

c. Compute the constant term and the first two harmonics in the Fourier series **(06 Marks)**

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

4 a. Find the Fourier Transform of

$$f(x)=\begin{cases}x & |x|\le\alpha\\ 0 & |x|>\alpha\end{cases}$$

Where α is a positive constant. **(07 Marks)**

b. Find the Fourier Cosine Transform of f(x)

$$f(x)=\begin{cases}4x & 0<x<1\\ 4-x & 1<x<4\\ 0 & x>4\end{cases}$$

(07 Marks)

c. Find Z-Transforms of Coshnθ and Sinhnθ. **(06 Marks)**

Contd... 2

(4)

MAT31

PART-C

5 a. Form a Partial differential equation by eliminating the arbitrary function ϕ from the equation $\phi(x+y+z, x^2+y^2-z^2)=0$ **(07 Marks)**

b. Solve $(y+z)p+(z+x)q = x+y$ **(07 Marks)**

c. Using Charpit's method, solve the equation

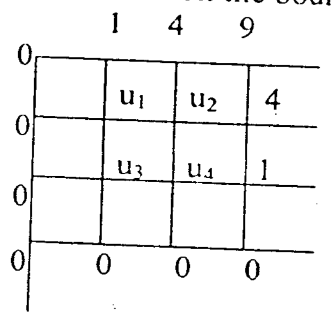
$$p^2+qy=z$$

(06 Marks)

6 a. Derive the one dimensional heat equation in usual form $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

b. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ for $0<x<1$, $0<y<1$ given that $u(x,0)=u(0,y)=0$, $u(x,1)=6x$ for $0\le x\le 1$ and $u(1,y)=3y$ for $0<y<1$. **(07 Marks)**

c. Evaluate the function u(x,y) satisfying $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ at the pivotal points given the values on the boundary as indicated in figure: **(06 Marks)**



PART-D

7 a. Show that the system of equations:

$$5x+y+3z=20$$
$$2x+5y+2z=18$$
$$3x+2y+z=14$$

is consistent and hence find the solution. **(07 Marks)**

b. Solve the following system of linear equations by Gauss-Siedel iterative method

$$10x+y+z=12$$
$$x+10y+z=12$$
$$x+y+10z=12$$

carry out 4 iteration. Starting with initial approximation $x_0=y_0=z_0=0$. **(07 Marks)**

c. Using the power method, find the dominant Eigen value and the corresponding Eigen vector of the matrix $A=\begin{bmatrix}25 & 1 & 2\\ 1 & 3 & 0\\ 2 & 0 & -4\end{bmatrix}$. Take $[1,0,0]^T$ as the initial vector. **(06 Marks)**

8 a. Obtain the Euler's equation in the form

$$\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0$$

(07 Marks)

b. Solve the Variational equation

$$\delta\int_0^{\pi/2}(y^2-y'^2)dx$$ under condition $y(0)=0$, $y(\pi/2)=2$ **(07 Marks)**

c. State and prove the Brachistochrone problem. **(06 Marks)**

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4 - copies

MAT31

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NEW SCHEME

Third Semester B.E. Degree Examination, Dec.06 / Jan.07

Common to All Branches

Engineering Mathematics - III

Time: 3 hrs.] [Max. Marks:100

Note: ***1. Answer any FIVE full questions choosing at least one from each part.***

Part - A

1 a. Using Regula-Falsi method compute the real root of the equation $x^3 - 4x - 9 = 0$ correct to two decimal places. **(06 Marks)**

b. The population of a town is given by the table :

Year	1951	1961	1971	1981	1991
Population in thousand	19.96	39.65	58.81	77.21	94.61

Using Newtons-Forward and Backward interpolation formula, calculate the increase in the population from the year 1955 to 1985. **(07 Marks)**

c. Construct an interpolating polynomial for the data given below using Newtons divided difference formula : **(07 Marks)**

x	2	4	5	6	8	10
$f(x)$	10	96	196	350	868	1746

2 a. Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ using Simpsons $\frac{3}{8}^{th}$ rule by considering seven ordinates and hence find the approximate value of π. **(06 Marks)**

b. Given that $\frac{dy}{dx} = x + y^2$ and $y(0) = 1$ find $y(0.2)$ in step size of 0.1 using modified Euler's method. Carry out two iterations in each step. **(07 Marks)**

c. From the data given below find y at $x = 1.4$ using Milne's predictor corrector formula. $\frac{dy}{dx} = x^2 + \frac{y}{2}$.

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514

(Use corrector formula twice) **(07 Marks)**

Part – B

3 a. An alternating current after passing through a half wave rectifier has the form,

$$i = \begin{cases} I_0 \sin\theta & \text{for } 0 \le \theta \le \pi \\ 0 & \text{for } \pi \le \theta \le 2\pi \end{cases}$$ (where I_0 in the maximum current).

Express i in a Fourier series. **(06 Marks)**

b. Obtain the Half-range cosine series for f(x) defined by,

$$f(x) = \begin{cases} K_x & \text{for } 0 \le x \le \frac{l}{2} \\ K(l-x) & \text{for } \frac{l}{2} \le x \le l \end{cases}$$ and hence find the sum of the series

$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots$ **(07 Marks)**

Contd....2

3 c. Express 'y' in a Fourier series up to second Harmonics given,

x^0	0	30	60	90	120	150	180	210	240	270	300	330	360
y	1.80	1.10	0.30	0.16	0.50	1.30	2.16	1.25	1.30	1.52	1.76	2.00	1.80

(07 Marks)

4 a. Find the Fourier transform of the function $f(x) = \begin{cases} a^2 - x^2 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$. (06 Marks)

b. Find the Fourier sine transform of $e^{-|x|}$ and hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}$ $m > 0$. (07 Marks)

c. Obtain the Z-transforms of $\cos n\theta$ and $\sin n\theta$. (07 Marks)

Part – C

5 a. Form the partial differential equation by eliminating arbitrary function from the equation $Z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. (06 Marks)

b. Solve $x^2(y-z)\frac{\partial z}{\partial x} + y^2(z-x)\frac{\partial z}{\partial y} = z^2(x-y)$. (07 Marks)

c. Solve $(p^2 + q^2)y = qz$, using Charpits method. (07 Marks)

6 a. Under suitable assumptions derive one dimensional wave equation in the form $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. (08 Marks)

b. Solve one dimensional heat equation $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$ under the following conditions.

i) $u(0,t) = u(4,t) = 0, t > 0$

ii) $u(x,0) = x(4-x)$ $(0 < x < 4)$.

Using Explicit formula by taking $u = 1$ and for $(0 < t \le 1)$ (12 Marks)

Part – D

7 a. Test for consistency and solve $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$. (06 Marks)

b. Apply Gause-Seidel iterative method to find the third approximate solution of the system of equations, $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$ with initial approximate solution (1, 0, 3). (07 Marks)

c. Use Rayleighs power method to find the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by taking initial eigen vector as $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

(Carry out 4 iterations). (07 Marks)

8 a. With usual notation derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. (06 Marks)

b. Find the curve passing through the points $(x_1 y_1)$ and $(x_2 y_2)$ which when rotated about x-axis gives a minimum surface area. (07 Marks)

c. Find the external of the functional,

$\int_{x_0}^{x_1} (x + y')y' dx$. (07 Marks)

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USN | | | | | | | | | |

Srinivas Institute of Technology

06MA131

Third Semester B.E. Degree Examination, Dec. 07 / Jan. 08
Engineering Mathematics - III

Time: 3 hrs. Max. Marks:100

Note : *Answer any FIVE full questions choosing at least TWO from each part.*

Part A

1 a. Find the Fourier series for the function $f(x)=x+x^2$ from $x=-\pi$ to $x=\pi$ and deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots\ldots$ **(07 Marks)**

b. Obtain the cosine half-range Fourier series for $f(x)=\begin{cases} Kx, & \text{in } 0\le x\le l/2 \\ K(l-x) & \text{in } l/2\le x\le l \end{cases}$. **(07 Marks)**

c. The following table gives the variating of periodic current over a period:

t (sec)	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
A (Amp)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic. **(06 Marks)**

2 a. Obtain the finite Fourier Cosine transform of the function $f(x)=e^{ax}$ in $(0, l)$. **(07 Marks)**

b. Find the Fourier sine and cosine transforms of

$$f(x)=\begin{cases} x, & 0<x<2 \\ 0, & \text{otherwise} \end{cases}.$$ **(07 Marks)**

c. Solve the integral equation,

$$\int_0^\infty f(x)\cos(\alpha x)dx=\begin{cases} 1-\alpha, & 0\le\alpha\le 1 \\ 0, & \alpha>1 \end{cases}.$$

Hence evaluate $\int_0^\infty \frac{1-\cos x}{x^2}dx$. **(06 Marks)**

3 a. Form the P.D.E by eliminating the arbitrary function from $z=y^2+2f\left(\frac{1}{x}+\log y\right)$. **(07 Marks)**

b. Solve $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}=0$ by the method of separation of variables. **(07 Marks)**

c. Solve $(y^2+z^2)p+x(yq-z)=0$. **(06 Marks)**

4 a. Derive the one dimensional heat equation. **(07 Marks)**

b. Solve the wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ given $u(0,t)=0$; $u(l,t)=0$; $\frac{\partial u}{\partial t}=0$ when $t=0$ and $u(x,0)=u_0\sin\frac{\pi x}{l}$. **(07 Marks)**

c. Obtain the various possible solutions of the Laplace's equation $u_{xx}+u_{yy}=0$ by the method of separation of variables. **(06 Marks)**

Part B

5 a. Find the real root of the equation $3x = \cos x + 1$ correct to four decimal places using Newton's method. **(07 Marks)**

b. Solve the system of equations,

$2x + y + z = 10$

$3x + 2y + 3z = 18$

$x + 4y + 9z = 16$

by Gauss-Jordan method. **(07 Marks)**

c. Find the largest eigen value and the corresponding eigen vector of the following matrix by using power method.

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Taking $[1 \quad 0 \quad 0]^T$ as the initial eigen vector. Carry out four iterations. **(06 Marks)**

6 a. Given f(0) = 1, f(1) = 3, f(2) = 7, f(3) = 13. Find f(0.1) and f(2.9) using Newton Interpolation formula. **(07 Marks)**

b. Using Newton's divided difference formula evaluate f(8) and f(15), given that **(07 Marks)**

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

c. Evaluate $\int_4^{5.2} \log_e x dx$ by using Weddle's rule, taking 7 ordinates. **(06 Marks)**

7 a. Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(07 Marks)**

b. Find the extremal of the functional $\int_0^{\pi/2} \left[y^2 - (y')^2 - 2y\sin x\right] dx$ under the conditions $y(0) = y(\pi/2) = 0$. **(07 Marks)**

c. Find the geodesics on a surface, given that the arc length on the surface is $s = \int_{x_1}^{x_2} \sqrt{x\left[1 + (y')^2\right]}dx$. **(06 Marks)**

8 a. Find the z-transforms of i) $(n+1)^2$ ii) sin(3n+5). **(07 Marks)**

b. Obtain the inverse Z transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. **(07 Marks)**

c. Solve the difference equation,

$y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z transforms. **(06 Marks)**

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06MAT31

Third Semester B.E. Degree Examination, June / July 08
Engineering Mathematics - III

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions choosing atleast TWO full questions from each part.***

PART – A

1 a. Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \le x \le \pi$. **(07 Marks)**

b. Obtain a half range cosine for **(07 Marks)**

$$f(x) = \begin{cases} k\,x & \text{for } 0 \le x \le l/2 \\ k(l-x) & \text{for } l/2 \le x \le l. \end{cases}$$

c. Obtain the constant term and the coefficients of the first sine and cosine terms in the Fourier expansion of y as given in the following table. **(06 Marks)**

x :	0	1	2	3	4	5
Y :	9	18	24	28	26	20

2 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2 & \text{if } |x| \le 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$ and use it to evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\frac{x}{2}\,dx$.

(07 Marks)

b. Find the Fourier Cosine transform of e^{-x^2}. **(07 Marks)**

c. Using convolution theorem, find the inverse Fourier transform of $H(\alpha) = \frac{1}{(1+\alpha^2)^2}$.

(06 Marks)

3 a. Form the partial differential equation by eliminating the arbitrary functions $F(x+2y) + G(x-3y) = 0$. **(07 Marks)**

b. Use the separation of variable technique to solve $3\,U_x + 2\,U_y = 0$. Given $U(x, 0) = 4\,e^{-x}$. **(07 Marks)**

c. Solve $(x^2 - y^2 - z^2)\,p + 2xy\,q = 2xz$. **(06 Marks)**

4 a. Derive the one dimensional wave equation in the standard form. **(06 Marks)**

b. Obtain the various solutions of the Laplace's equation $U_{xx} + U_{yy} = 0$ by the method of separation of variables. **(07 Marks)**

c. A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set to vibrate by giving each point a velocity $V_0 \sin^3 \frac{\pi x}{l}$. Find the displacement $y(x, t)$. **(07 Marks)**

PART – B

5 a. Compute the real root of the equation $x \log_{10} x - 1.2 = 0$ correct to five decimal places using Regula Falsi method. **(07 Marks)**

b. Solve the following system of equations by Gauss – Seidel iteration method. **(07 Marks)**
$27x + 6y - z = 85$, $6x + 15y + 2z = 72$, $x + y + 54z = 110$.

c. Find the largest Eigen value and the corresponding Eigen vector of the following matrix by using power method : $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$. Take $(1\ 0\ 0)^T$ as the initial Eigen vector. Carry out 4 iterations. **(06 Marks)**

6 a. Use Newton's divided difference formula to find f(8) given. **(07 Marks)**

x :	4	5	7	10	11	13
f(x) :	48	100	294	900	1210	2028

b. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.05 given **(07 Mark.**

x : 1	1.05	1.1	1.15	1.2	1.25	1.3
f(x) : 1	1.0247	1.04881	1.07238	1.09544	1.11803	1.14017

c. By Dividing the range into 6 equal parts, find the approximate value of $\int_0^{\pi} e^{\sin x}\, dx$ using simpsons $1/3^{rd}$ rule. **(06 Marks)**

7 a. Derive Euler's equation in the form $\frac{d}{dx}\left(f - y'\frac{\partial f}{\partial y'}\right) - \frac{\partial f}{\partial x} = 0$. **(07 Marks)**

b. Find the extremal of the function $\int_0^{\pi/2} (y'^2 - y^2 + 4y\cos x)\,dx$ given $y(0) = 0$, $y(\pi/2) = 0$. **(06 Mar.**

c. Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about the x – axis gives a minimum surface area. **(07 Marks)**

8 a. Find the Z – transform of i) n^2 ii) $\cos n\theta$. **(07 Marks)**

b. Find the inverse Z – transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. **(07 Marks)**

c. Solve the difference equation $Y_{n+2} + 2Y_{n+1} + Y_n = n$ with $Y_0 = Y_1 = 0$, using Z – transforms. **(06 Marks)**

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06MAT31

Third Semester B.E. Degree Examination, June-July 2009
Engineering Mathematics-III

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Obtain Fourier series for the function

$$f(x)=\begin{cases}\pi x & \text{for } 0\le x\le 1\\ \pi(2-x) & \text{for } 1\le x\le 2\end{cases}$$ and hence deduce that $$\frac{\pi^2}{8}=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$$ (07 Marks)

b. Obtain the half range cosine series for the function $f(x) = \sin x$ in $0 \le x \le \pi$. (07 Marks)

c. Express y as a Fourier series up to first harmonics given

x :	0	60°	120°	180°	240°	300°	360°
y :	7.9	7.2	3.6	0.5	0.9	6.8	7.9

(06 Marks)

2 a. Find the Fourier transform of

$$f(x)=\begin{cases}1 & \text{for } |x|<1\\ 0 & \text{for } |x|>1\end{cases}$$ Hence evaluate $\int_0^{\infty}\frac{\sin x}{x}dx$ (07 Marks)

b. Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$ (07 Marks)

c. Solve the integral equation $\int_0^{\infty} f(\theta)\cos\alpha\theta\, d\theta = \begin{cases}1-\alpha, & 0\le\alpha\le 1\\ 0, & \alpha>1\end{cases}$ Hence evaluate $\int_0^{\infty}\frac{\sin^2 t}{t^2}dt$ (06 Marks)

3 a. Find the partial differential of all planes which are at constant distance from the origin. (07 Marks)

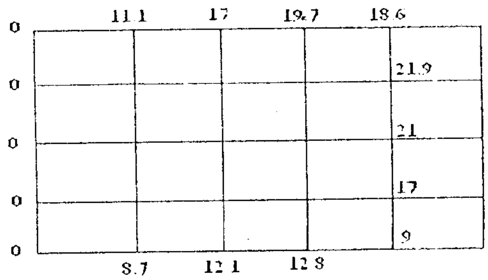
b. Using the method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$ (07 Marks)

c. Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$ (06 Marks)

4 a. Derive one dimensional heat equation. (07 Marks)

b. Obtain D'Alembert's solution of wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ (07 Marks)

c. Solve the Laplace's equation $U_{xx} + U_{yy} = 0$ given that



(06 Marks)

PART – B

5 a. Using Newton-Raphson method find the real root of the equation $3x = \cos x + 1$ (07 Marks)

b. Solve the following system of equations using Gauss-Jordon method

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

(07 Marks)

c. Find the largest eigen value and the corresponding eigen vector of the following matrix by using power method

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$ Take $(1, 0, 0)^T$ as initial eigen vector. Carry out four iterations. (06 Marks)

6 a. A slider in a machine moves along a fixed straight rod. Its distance *x* cm along the rod is given below for various values of the time *t* sec. Find the velocity and its acceleration when $t = 0.3$ sec.

t	0	0.1	0.2	0.3	0.4	0.5
x	30.13	31.62	32.87	33.64	33.95	33.81

(07 Marks)

b. Given the values of x and y

x :	1.2	2.1	2.8	4.1	4.9	6.2
y :	4.2	6.8	9.8	13.4	15.5	19.6

Find the value of x corresponding to y = 12 using Lagrange's technique. (07 Marks)

c. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Weddle's rule taking 7 ordinates. (06 Marks)

7 a. Find the extremal of the functional $\int_0^1 \left[(y')^2 + 12xy\right]dx$ with $y(0)=0$ and $y(1) = 1$. (07 Marks)

b. Find the curve passing through the points $(x_1\ y_1)$ and $(x_2\ y_2)$ which when rotated about the x-axis gives a minimum surface area. (07 Marks)

c. Show that the geodesics on a plane are straight lines. (06 Marks)

8 a. Find the Z-transform of the following:

i) $(n + 1)^2$

ii) $5m\,(3n + 5)$

(07 Marks)

b. Find the inverse Z-transform of $\frac{z^3 - 20z}{(z-2)^3(z-4)}$ (07 Marks)

c. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z-transforms. (06 Marks)

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Third Semester B.E. Degree Examination, Dec.09/Jan.10
Engineering Mathematics - III

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Obtain Fourier series for the function f(x) given by

$$f(x) = \begin{cases} 1+\dfrac{2x}{\pi}, & -\pi \le x \le 0 \\ 1-\dfrac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$$

and hence deduce that $\dfrac{\pi^2}{8} = \dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\ldots\ldots\ldots\ldots$ **(07 Marks)**

b. Find the half-range cosine series for the function $f(x) = (x-1)^2$ in the interval $0 < x < 1$. **(07 Marks)**

c. Obtain the constant term and the coefficients of the first sine and cosine terms in the Fourier expansion of y as given in the following table. **(06 Marks)**

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

2 a. Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & |x| \le 0 \\ 0, & |x| > \pi \end{cases}$.

Hence evaluate $\displaystyle\int_0^\infty \frac{x\cos x - \sin x}{x^3}.\cos\left(\frac{x}{2}\right)dx$ **(07 Marks)**

b. Find the Fourier cosine transform of e^{-x^2} **(07 Marks)**

c. Find the Fourier sine transform of $e^{-|x|}$. Hence show that $\displaystyle\int_0^\infty \frac{x\sin mx}{1+x^2}dx = \frac{\pi}{2}.e^{-m}$, $m > 0$. **(06 Marks)**

3 a. Form the partial differential equation by eliminating the arbitrary functions f and g from the relation $z = y^2 + 2f\left(\dfrac{1}{x}+\log y\right)$ **(07 Marks)**

b. Solve $\dfrac{\partial^3 t}{\partial x^2 \partial y} + 18xy^2 + \sin(2x-y) = 0$ **(07 Marks)**

c. Solve $(mz-ny)\dfrac{\partial z}{\partial x} + (nx-lz)\dfrac{\partial z}{\partial y} = (ly-mx)$ **(06 Marks)**

4 a. Derive one dimensional heat equation by taking u(x, t) as the temperature, x is the distance and t is the time. (Write the figure also.) **(07 Marks)**

b. Obtain the D'Almbert's solution of the wave equation $u_{tt} = C^2 u_{xx}$, subject to the condition $u(x, o) = f(x)$ and $\dfrac{\partial u}{\partial t}(x, o) = o$. **(07 Marks)**

c. Obtain the various solutions of the Laplace's equation $u_{xx} + u_{yy} = 0$, by the method of separation of variables. **(06 Marks)**

PART – B

5 a. Complete the real root of the equation $x\log_{10}x - 1.2 = 0$ by Regula-Falsi method, correct to four decimal places. **(07 Marks)**

b. Solve the system of equations using Gauss-Jordan method:

$$2x + 5y + 7z = 52$$
$$2x + y - z = 0$$
$$x + y + z = 9$$

(07 Marks)

c. Using the power method, find the largest Eigen value and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **(06 Marks)**

6 a. The area of a circle (A) corresponding to diameter (D) is given below: **(07 Marks)**

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 105 using an appropriate interpolation formula.

b. Use Newton's divided difference formula to find f(9), given the data, **(07 Marks)**

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

c. Evaluate $\int_4^{5.2} \log_e x \, dx$ using Weddle's rule, taking 7 ordinates. **(06 Marks)**

7 a. Derive Euler's equation in the form

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

(07 Marks)

b. Find the curves on which the functional $\int_0^1 [(y')^2 + 12xy] dx$, with $y(0) = 0$ and $y(1) = 1$ can be extremised. **(07 Marks)**

c. Find the geodesics on a surface given that the arc length on the surface is $S = \int_{x_1}^{x_2} \sqrt{x[1 + (y')^2]}\, dx$ **(06 Marks)**

8 a. Find the Z-transforms of the following :

i) $\cosh n\theta$ ii) $(n + 1)^2$ **(07 Marks)**

b. Find the inverse z-transform of $\frac{z}{(z-1)(z-2)}$. **(07 Marks)**

c. Find the response of the system $y_{n+2} - 5y_{n+1} + 6y_n = u$, with $y_0 = 0$, $y_1 = 1$ and $u_n = 1$ for n = 0, 1, 2, 3, ... by z-transform method. **(06 Marks)**

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06MAT31

Third Semester B.E. Degree Examination, May/June 2010
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Expand the function $f(x) = x - x^2$ in the interval $-\pi < x < \pi$. Deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots$ **(07 Marks)**

b. Find the half-range cosine series for the function $f(x) = (x-1)^2$ in $0 < x < 1$. **(07 Marks)**

c. The following table gives the variations of periodic current over a period

t (sec) :	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
A (amp) :	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the 1st harmonic. **(06 Marks)**

2 a. Express the function $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ as a Fourier integral and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(07 Marks)**

b. Find Fourier sine transform of $\frac{1}{x} e^{-ax}$. **(07 Marks)**

c. Use convolution theorem to find the inverse Fourier transform of $\frac{1}{(1+s^2)^2}$ given that $\frac{2}{1+s^2}$ is the Fourier transform of $e^{-|x|}$. **(06 Marks)**

3 a. Form the partial differential equation by eliminating the arbitrary function from $Z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. **(07 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, given that $\frac{\partial z}{\partial x} = -2 \sin y$, when $x = 0$; and $z = 0$ when y is an odd multiple of $\pi/2$. **(07 Marks)**

c. Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. **(06 Marks)**

4 a. Derive the one dimensional heat equation in the standard form. **(07 Marks)**

b. Obtain the various solutions of the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ by the method of separation of variables. **(07 Marks)**

c. A string stretched between the two fixed points (0, 0) and (1, 0) and released at rest from the position $y = \lambda \sin(\pi x)$. Show that the formula for its subsequent displacement y (x, t) is $\lambda \cos(c\pi t) \sin(\pi x)$. **(06 Marks)**

PART – B

5 a. Show that a real root of the equation tan x + tan hx = 0 lies between 2 and 3. Then apply the regula falsi method to find the third approximation. (07 Marks)

b. Apply Gauss – Jordan method to solve the system of equations:
$2x + 5y + 7z = 52$; $2x + y - z = 0$; $x + y + z = 9$. (07 Marks)

c. Use power method to find the dominant eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ with the initial eigen vector as $[1, 1, 1]^T$. (06 Marks)

6 a. Under the suitable assumptions find the missing terms in the following table:

x :	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
f (x) :	2.6	-	3.4	4.28	-	14.2	29

(07 Marks)

b. Use Newton's divided difference formula to find f (4) given :

x :	0	2	3	6
f (x) :	-4	2	14	158

(07 Marks)

c. Using Simpson's $\frac{3}{8}^{th}$ rule, evaluate $\int_0^{0.3} \sqrt{1-8x^3}\, dx$, by taking 7 ordinates. (06 Marks)

7 a. Solve the variational problem $\delta \int_0^{\pi/2} \left((y)^2 - (y')^2\right) dx$ under the conditions $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 2$. (07 Marks)

b. Find the curve on which the function $\int_0^{\pi/2} \left[(y)^2 - (y')^2 - y \sin x\right] dx$ under the conditions $y(0) = y\left(\frac{\pi}{2}\right) = 0$, can be extremised. (07 Marks)

c. Prove that the catenary is the plane curve which when rotated about a line (x – axis) generates a surface of revolution of minimum area. (06 Marks)

8 a. Find the Z – transform of i) n^2 ; ii) $n\, e^{-an}$. (07 Marks)

b. Prove that : i) $Z(\cos n\theta) = \dfrac{z(z - \cos\theta)}{z^2 - 2z\cos\theta + 1}$; ii) $z(\sin n\theta) = \dfrac{z \sin\theta}{z^2 - 2z\cos\theta + 1}$. (07 Marks)

c. Find the inverse Z – transform of $\dfrac{Z}{(Z-1)(Z-2)}$. (06 Marks)

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